## ПATIBIA UПIVERSITY OF SCIEПCE AПD TECHПOLOGY

## FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: BACHELOR OF SCIENCE HONOURS IN APPLIED STATISTICS |  |
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| QUALIFICATION CODE: <br> O8BSHS | LEVEL: 8 |
| COURSE CODE: SAT802S | COURSE: SAMPLING THEORY |
| SESSION: JANUARY 2023 | PAPER : THEORY |
| DURATION: 3 Hours | MARKS: 100 |


| SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER |  |
| :--- | :---: |
| EXAMINER | Mr. J. J. SWARTZ |
| MODERATOR: | Dr. I. NEEMA |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Calculator
2. Pen and Clean Paper for calculations

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

## Question 1 [25 marks]

1.1. What is meant by the sampling distribution of a statistic?
1.2. Select all the 20 samples of size three from the population of six students in Table 2.1, below without replacement. From each sample, find the $95 \%$ confidence limits for the population mean of the math scores with the known population variance and its estimates; use the normal deviate $Z=1.96$ in both cases.
1.2.1 For both the procedures, find the proportion of the confidence intervals enclosing the actual population mean, that is, the coverage probability.
1.2.2 Compare the average of the confidence widths obtained with the estimates of variance with the exact width for the case of known variance.
table 2.1. SAT verbal and math scores.

| Student | Verbal | Math |
| :---: | :---: | :---: |
| $i$ | $x_{i}$ | $y_{i}$ |
| 1 | 520 | 670 |
| 2 | 680 | 720 |
| 3 | 500 | 650 |
| 4 | 580 | 720 |
| 5 | 530 | 560 |
| 6 | 480 | 700 |
| Total | 3300 | 4020 |
| Mean | 550 | 670 |
| Variance |  |  |
| $\sigma^{2}$ | 4866.67 | 3066.67 |
| $S^{2}$ | 5840 | 3680 |
| $S$ | 76.42 | 60.66 |
| C.V. (\%) | 13.89 | 9.05 |

c.V. $=$ coefficient of variation.

## Question 2 [28 marks]

2.1. Provide and explain four basic criteria for the acceptability of a sampling method?
2.2. The investigator samples 10 one-acre plots by simple random sampling and counts the number of trees ( $y$ ) on each plot. She also has aerial photographs of the plantation from which she can estimate the number of trees ( $x$ ) on each plot of the entire plantation. Hence, she knows $\mu_{x}=19.7$ and since the two counts are approximately proportional through the origin, she uses a ratio estimate to estimate $\mu_{y}$
Table 1: To estimate the average number of trees per acre on a 1000 - acre plantation

| Plot | Actual no. per acre Y | Aerial estimate X | $y_{i}-r x_{i}$ |
| :---: | :---: | :---: | :---: |
| 1 | 25 | 23 | 0.5625 |
| 2 | 15 | 14 | 0.1250 |
| 3 | 22 | 20 | 0.7500 |
| 4 | 24 | 25 | -2.5625 |
| 5 | 13 | 12 | 0.2500 |
| 6 | 18 | 18 | -1.1250 |
| 7 | 35 | 30 | 3.1250 |
| 8 | 30 | 27 | 1.3125 |
| 9 | 10 | 8 | 1.5000 |
| 10 | 29 | 31 | -3.9375 |
| mean | 22.10 | 20.80 | - |



Figure 1: Scatter plot

The segression equetson is
鿊 $=3.24+1.06 \mathrm{x}$

|  | Cวะを | 4\% Coez | I |
| :---: | :---: | :---: | :---: |
| Cometant | 1.236 | 2.003 | 0. 82 |

$\mathrm{x} \quad 2.00293 \quad 0.09094 \quad 4.03 \quad 0.000$

Figure 2: Regression output
2.2.2. Construct the approximate $95 \%$ confidence interval for $\mu_{y}$

## Question 3 [17 marks]

3.1. The New York Times of February 25, 1994, summarized the results of a survey conducted by Klein Associates, Inc. on 2000 lawyers on sexual advances in the office. Between 85 and $98 \%$ responded to the questions in the survey; $49 \%$ of the responding women and $9 \%$ of the responding men agreed that some sorts of harassment exist in the offices. Assume that the population of lawyers is large and there are equal numbers of female and male lawyers, and ignore the nonresponse; that is, consider the respondents to be a random sample of the 2000 lawyers.
3.1.1 Find the standard errors for the above percentages.
3.2. To estimate the percentage of people that carries a viral infection which produces AIDS, 128 people are examined and 72 of them are found to be infected. Calculate the standard error of the estimated proportion and compute a $95 \%$ confidence interval for the population proportion?
3.3. If no information of P (proportion) is provided when determining the sample size of a population, find the error of the estimation e for $\mathrm{n}=2000$, Consider $\alpha=0.05$ for both cases.

## Question 4 [30 marks]

4.1.

Let there exist a population $U=\{1,2,3\}$ with the following design:

$$
p(\{1,2\})=\frac{1}{2}, p(\{1,3\})=\frac{1}{4}, p(\{2,3\})=\frac{1}{4} .
$$

Give the first-order inclusion probabilities. Give the variance-covariance matrix $\Delta$ of indicator variables for inclusion in the sample. Give the variance matrix of the unbiased estimator for the total.
4.2. Between the 100 computer corporations in Namibia, the average of employee sizes for the largest 10 and smallest 10 corporations were known to be 300 and 100, respectively.

For a sample of 20 from the remaining 80 corporations, the mean and standard deviation were 250 and 110 , respectively. For the total employee size of the 80 corporations, find the
a) Estimate,
b) S.E. of the estimate, and
c) $95 \%$ confidence limits.

4.3. Write a short description on the importance of the normal distribution in sampling
theory
4.4. Write at least 4 properties of the normal probability distribution.

